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June 2009
6681 Mechanics M5
Mark Scheme

Question Number	Scheme	Marks
Q1	$\pm(8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k})$	B1
	$((4\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}) + (8\mathbf{i} - 4\mathbf{j} + 7\mathbf{k})).(8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k}) = \frac{1}{2}3v^2$	M1 A1 f.t.
	$12 = v$	A1
	$\mathbf{v} = \frac{12}{\sqrt{8^2 + (-4)^2 + 8^2}}(8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k})$	M1
	$\mathbf{v} = (8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k})\,\text{ms}^{-1}$	DM1 A1
		[7]
Q2	C.F. is $\mathbf{r} = \mathbf{A}\cos 2t + \mathbf{B}\sin 2t$	B1
	P.I. is $\mathbf{r} = \mathbf{p}e^{2t}$	B1
	$\dot{\mathbf{r}} = 2\mathbf{p}e^{2t}$ $\ddot{\mathbf{r}} = 4\mathbf{p}e^{2t}$	B1 ft
	$4\mathbf{p}e^{2t} + 4\mathbf{p}e^{2t} = \mathbf{j}e^{2t}$	M1
	so, (PI is) $\mathbf{r} = \frac{1}{8}\mathbf{j}e^{2t}$	A1
	GS is $\mathbf{r} = \mathbf{A}\cos 2t + \mathbf{B}\sin 2t + \frac{1}{8}\mathbf{j}e^{2t}$	A1 ft
	$t = 0,\ \mathbf{r} = \mathbf{i} + \mathbf{j} \Rightarrow \mathbf{i} + \mathbf{j} = \mathbf{A} + \frac{1}{8}\mathbf{j} \Rightarrow \mathbf{i} + \frac{7}{8}\mathbf{j} = \mathbf{A}$	DM1 A1
	$\dot{\mathbf{r}} = -2\mathbf{A}\sin 2t + 2\mathbf{B}\cos 2t + \frac{1}{4}\mathbf{j}e^{2t}$	M1A1
	$t = 0,\ \dot{\mathbf{r}} = 2\mathbf{i} \Rightarrow 2\mathbf{i} = 2\mathbf{B} + \frac{1}{4}\mathbf{j} \Rightarrow \mathbf{i} - \frac{1}{8}\mathbf{j} = \mathbf{B}$	A1
	$\mathbf{r} = (\mathbf{i} + \frac{7}{8}\mathbf{j})\cos 2t + (\mathbf{i} - \frac{1}{8}\mathbf{j})\sin 2t + \frac{1}{8}\mathbf{j}e^{2t}$	**[11]**

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Question Number	Scheme	Marks
Q3 (a)	$mv = (m+\delta m)(v+\delta v)-(-\delta m)(c-v)$ $mv = mv + m\delta v + v\delta m + c\delta m - v\delta m$ $-m\delta v = c\delta m$ $\frac{\mathrm{d}v}{\mathrm{d}m} = -\frac{c}{m}$ *	M1 A2 DM1 A1 (5)
(b)	$\frac{\mathrm{d}m}{\mathrm{d}t} = -m_0 k$	B1
	$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}v}{\mathrm{d}m} \times \frac{\mathrm{d}m}{\mathrm{d}t}$ $= -\frac{c}{m} \times -m_0 k$ $= \frac{cm_0 k}{m_0(1-kt)}$ $= \frac{ck}{(1-kt)}$	M1 DM1 A1 (4) **[9]**

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Question Number	Scheme	Marks
Q4 (a)	$\delta m = \dfrac{2Mx\delta x}{a^2}$	M1 A1
	$\delta I = \dfrac{1}{3}\dfrac{2Mx\delta x}{a^2}(2x)^2$	M1 A1
	$I = \int_0^a \dfrac{8Mx^3 \mathrm{d}x}{3a^2}$	DM1
	$= \dfrac{8M}{3a^2}\left[\dfrac{x^4}{4}\right]_0^a$	
	$= \dfrac{2}{3}Ma^2$ *	A1
		(6)
(b)	$J.2a = \dfrac{2}{3}Ma^2\omega$	M1 A1
	$\dfrac{1}{2}\dfrac{2}{3}Ma^2\omega^2 = Mg\dfrac{2a}{3}(1+\cos 60^o)$	M1 A2
	solving for J	DM1
	$J = M\sqrt{\dfrac{ag}{3}}$	A1 (7)
		[13]

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Question Number	Scheme	Marks
Q5 (a)	$(2\mathbf{i} + \mathbf{j}) + (-2\mathbf{j} - \mathbf{k}) + \mathbf{F}_3 = \mathbf{0}$	M1
	$\mathbf{F}_3 = -2\mathbf{i} + \mathbf{j} + \mathbf{k}$	A1
	$\lvert\mathbf{F}_3\rvert = \sqrt{(-2)^2 + 1^2 + 1^2} = \sqrt{6}$ N	M1 A1 (4)
(b)	$(3\mathbf{i} + \mathbf{j} + \mathbf{k}) \times (2\mathbf{i} + \mathbf{j}) + (\mathbf{i} - 2\mathbf{j}) \times (-2\mathbf{j} - \mathbf{k}) + (x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \times (-2\mathbf{i} + \mathbf{j} + \mathbf{k})$	M1
	$(-\mathbf{i} + 2\mathbf{j} + \mathbf{k}) + (2\mathbf{i} + \mathbf{j} - 2\mathbf{k}) + ((y - z)\mathbf{i} + (-2z - x)\mathbf{j} + (x + 2y)\mathbf{k})$	A3
	$y - z = -1,\ -x - 2z = -3,\ x + 2y = 1$	DM1
	$x = 1,\ y = 0,\ z = 1$ is a solution	DM1
	so, $\mathbf{r} = (\mathbf{i} + \mathbf{k}) + \lambda(-2\mathbf{i} + \mathbf{j} + \mathbf{k})$ is a vector equn of line of action of $\mathbf{F}_3$	M1 A1 (8)
(c)	$(3\mathbf{i} + \mathbf{j} + \mathbf{k}) \times (2\mathbf{i} + \mathbf{j}) + (\mathbf{i} - 2\mathbf{j}) \times (-2\mathbf{j} - \mathbf{k}) = \mathbf{G}$	M1
	$(-\mathbf{i} + 2\mathbf{j} + \mathbf{k}) + (2\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = (\mathbf{i} + 3\mathbf{j} - \mathbf{k}) = \mathbf{G}$	A1
	$\lvert\mathbf{G}\rvert = \sqrt{1^2 + 3^2 + (-1)^2} = \sqrt{11}$ N m	M1 A1 (4)
		[16]

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Question Number	Scheme	Marks
Q6 (a)	$\frac{1}{3}2m(4a)^2+\frac{1}{12}4ma^2+4m(4a)^2$	B1 M1 A1
	$=\frac{32}{3}ma^2+\frac{1}{3}ma^2+64ma^2$	
	$=75ma^2$ *	A1 (4)
(b)	$\frac{1}{2}75ma^2\omega^2=2mg2a(\cos\theta-\cos\alpha)+4mg4a(\cos\theta-\cos\alpha)$	M1 A2
	$a\omega^2=\frac{8}{15}g(\cos\theta-\frac{24}{25})=\frac{8}{375}g(25\cos\theta-24)$	A1
	$X-6mg\cos\theta=2m2a\omega^2+4m4a\omega^2=20ma\omega^2$	M1 A2
	$X=6mg\cos\theta+20m\frac{8}{375}g(25\cos\theta-24)$	D M1
	$=\frac{50mg\cos\theta}{3}-\frac{256mg}{25}$	A1 (9)
(c)	$-2mg2a\sin\theta-4mg4a\sin\theta=75ma^2\ddot{\theta}$	M1 A1
	$\ddot{\theta}=-\frac{4g}{15a}\sin\theta$	A1
	$\approx-\frac{4g}{15a}\theta$, SHM	M1
	Time $=\frac{1}{4}2\pi\sqrt{\frac{15a}{4g}}$	M1
	$=\frac{\pi}{4}\sqrt{\frac{15a}{g}}$	A1 (6)
		[19]